

University of Bahrain
College of Science
Department of Mathematics
First Semester 2006/2007
Final Examination

Course: MATHS 102

Date: 15/01/2007

Time: 11:30 - 13:30

Name:

I.D. No.

Section:

PLEASE MAKE SURE YOUR COPY OF THIS EXAMINATION CONSISTS OF 7 DIFFERENT QUESTIONS.

Question	Points Possible	You Scored
1	12	
2	10	
3	12	
4	16	
5	21	
6	12	
7	17	
Total	100	

Question 2 [5+5 marks]

1) Derive the formula $1 - \tanh^2 x = \operatorname{sech}^2 x$

2) Find $\frac{dy}{dx}$ for $y = e^{\tanh x} - \sinh^{-1}(\ln x)$

Question 3 [6+6 marks]

Evaluate each of the following integrals

(i) $\int x^3 \sqrt{1+x^2} dx$

$$(ii) \int_0^{\pi/2} \cos^2 x \, dx$$

Question 4 [5+5+6 marks]

1) Evaluate $\lim_{x \rightarrow 0^+} \left(\cot x - \frac{1}{x} \right)$

2) Determine the convergence or divergence of

(i) $\left\{ \left(\cos \frac{1}{n} \right)^n \right\}$

(ii) $\int_0^1 \ln x \, dx$ (**Hint:** $\lim_{x \rightarrow 0^+} x \ln x = 0$)

Question 5 [6+5+5+5 marks]

- 1) Use the integral test to determine whether $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^{3/2}}$ is convergent or divergent.

(**Hint:** $\frac{d}{dx} \left(\frac{1}{x(\ln x)^{3/2}} \right) = \frac{-\frac{1}{2} \sqrt{\ln x} [2 \ln x + 3]}{x^2 (\ln x)^3}$).

- 2) Determine if the series is absolutely convergent, conditionally convergent or divergent.

(i)
$$\sum_{k=1}^{\infty} (-1)^k \frac{\sqrt[3]{k}}{(k^3 + 1)^{1/2}}$$

(ii)
$$\sum_{k=2}^{\infty} (-1)^k \frac{k^2 - 1}{(k + 1)!}$$

(iii)
$$\sum_{k=1}^{\infty} \frac{(-5)^k}{3^{k+1}}$$

Question 6 [5+7 marks]

1) Use a geometric series to find a power series representation of $f(x) = \frac{2}{1+4x^2}$.

2) Find the radius and interval of convergence of $\sum_{k=1}^{\infty} \frac{(-1)^k 4^k}{k} (x+1)^k$.

